Forward scattering of light in inhomogeneous binary dielectric media

Young Hoon Rim
Department of Physics, Semyung University, Chechon, Choongbuk 390-230, Korea
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We consider the forward scattering of light for large-scale inhomomogeneities compared to the wavelength in the binary dielectric medium. We calculate the scattering factor explicitly in a binary medium, based on the random-phase approximation of a scalar field equation. In such a discrete medium, the broadness of the full width at half maximum of the forward scattering intensity depends on the variance of local fluctuation of the dielectric constant. We discuss the intensity profiles of the forward scattering.

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The wave propagation and scattering theory in random media has applications in diverse areas of physics and applied science such as optical propagation in the atmosphere, acoustic scattering in the ocean, ionospheric scintillation, microwave scattering and remote sensing in a geophysical media, and wave propagation in radio astronomy [1–5]. The light scattering study has been successfully used for the characterization of materials with random structures not only for the solid samples such as polycrystalline materials but also for liquid samples like the colloidal particles [6–10].

In this paper, we present a theoretical study for forward scattering of light in a random discrete inhomogeneous dielectric medium with binary layers, which is the extended study of Rim, Haus, and Schroeder [6]. We consider the forward scattering of light for large-scale inhomogeneities compared to the wavelength in the binary dielectric medium. To analytically treat the inhomogeneity in this limit, however, we introduce a binary multislab model that partitions the medium into homogeneous thick and thin boxes representing different dielectric permittivities embedded in the medium. We solve for the fields using the random-phase screen [4,6,11] and we determine the intensity profile of the forward scattering radiation in terms of scattering factors. This profile has an angular distribution, which may then be compared with the previous results where we kept the average value of the correlation amplitude throughout the medium [6].

We shall assume that the local fluctuation $\delta\varepsilon(\mathbf{x})$ of the dielectric constant is a Gaussian stochastic variable. The correlation amplitude of the second moment of the stochastic variable represents the variance of local fluctuation of the dielectric constant, which is of the order of 10^{-2} for the polycrystalline samples [6]. In general, the dielectric constant is a function of frequency, thus the correlation amplitude also depends on the incident wavelength. If the scattered light is strongly correlated in the medium, the correlation length of the fluctuations may be much larger than the size of the inhomogeneities. The typical length of the correlation is 1 mm, which is 40 times the grain size of the polycrystalline samples [6].

If we assume that the local correlation amplitude of the dielectric constant and the thickness of the sample are the same throughout the multislab medium, the full width at half maximum (FWHM) of the forward scattering in the discrete multislab medium should be nearly the same as the one from the continuum in the limit of the line-of-sight propagation [6], since the interference effects from the multislab boundary are negligibly small for large-scale inhomogeneous forward scattering. We, however, note that the FWHM of the forward scattering intensity in the binary multislab medium depends on the value of the local correlation amplitude, which shows different characteristics from the same thickness layers medium. In the binary multislab medium, the FWHM is broadened if the local correlation amplitude is large in the thick layers of the binary medium and the FWHM changes negligibly for varying magnitude of the correlation amplitude in the thin layers. Correspondingly, the scattering intensity is decreased for strong local correlation amplitude in the thick layers through the propagation due to the strong absorbance.

The average is taken with respect to the stochastic variable $\delta\varepsilon(\mathbf{x})$ in the second moment approximation. That is, we shall assume that $\delta\varepsilon(\mathbf{x})$ is a Gaussian stochastic variable with the second moment correlation function having the form

$$\langle \delta\varepsilon(\mathbf{x})\delta\varepsilon(\mathbf{x}')\rangle = A \exp\left[-\frac{|\mathbf{x}-\mathbf{x}'|^2}{\sigma^2}\right], \quad (1)$$

where A is the correlation amplitude and σ is the correlation length in the medium. The correlation amplitude represents the variance of local fluctuations of the dielectric constant inside the medium, and the correlation length represents the scale of the correlated scattering distance through the medium. That is, $A=\langle \delta\varepsilon(\mathbf{x})\delta\varepsilon(\mathbf{x})\rangle = \langle \varepsilon^2(\mathbf{x})\rangle - \langle \varepsilon(\mathbf{x})\rangle^2$. Moreover, the variance of local fluctuations A plays an important role in a multislab medium to determine whether the medium has characteristics of strong or weak fluctuations. In a strong local fluctuation medium the correlation amplitude is very large, and the corresponding FWHM is broad, while in the weak fluctuation medium the correlation amplitude is relatively small and the corresponding FWHM is narrow. The strong local fluctuation of the dielectric constant inside the medium makes the correlation length long [6].

We have considered that the forward scattering field propagates in the sense of the Rytov method [3], which is widely used in the line-of-sight scattering process. This process is illustrated in Fig. 1. The ensemble average of the exponent of the stochastic variable $\delta\varepsilon$ can be expressed in terms of moments and cumulants. Using the characteristic functional of the process $\delta\varepsilon$ and second moment approximation [6], the forward scattering far-field intensity in the finite discrete binary medium, when $\rho \ll \sigma$, is

$$I_F(\theta)=|E_0|^2\frac{k_0^2}{R^2}(2\pi W)^2\frac{1}{2D}\exp\left[-\frac{k_0^2\sin^2\theta}{4D}\right], \tag{2}$$

where

$$D=\frac{1}{2W^2}+\frac{k^2}{4\langle\varepsilon\rangle^2\sigma^2}\left[\frac{N}{2}S_{0,\mathrm{odd}}\left[\frac{d}{\sigma}\right]+\frac{N}{2S_{0,\mathrm{even}}\left[\frac{d}{\sigma}\right]}+2\sum_{l=1}^{N/2}\{N-(2l-1)\}S_{2l-1}^{oe}\left[\frac{d}{\sigma}\right]+2\sum_{l=1}^{N/2-1}\left[\frac{N}{2}-1\right]\left[S_{2l}^{oo}\left[\frac{d}{\sigma}\right]+S_{2l}^{ee}\left[\frac{d}{\sigma}\right]\right]\right], \tag{3}$$

where k_0 is the wave number in free space, d represents a slab thickness, k is the wave number in the dielectric medium, $k=k_0\sqrt{\langle\varepsilon\rangle}=(2\pi/\lambda_0)\sqrt{\langle\varepsilon\rangle}$, and W is the incident beam width. The second or third term, i.e., $S_{0,\mathrm{odd}}$ or $S_{0,\mathrm{even}}$, in Eq. (3) is due to the self-correlation at the same site of the odd or even number layers. The fourth term represent the cross correlation at different sites of the odd-even number layers. Similarly, the fifth term in the parentheses (S_{2l}^{oo} and S_{2l}^{ee}) represent the cross correlations at different sites of the odd-odd number layers and even-even number layers, respectively. Thus the FWHM, Γ_F, of the forward scattering intensity in the finite discrete binary multislab medium is expressed by

$$\Gamma_F=2\sin^{-1}\left[\ln 2\left\{\frac{2}{k_0^2W^2}+\frac{1}{\langle\varepsilon\rangle\sigma^2}\left[\frac{N}{2}S_{0,\mathrm{odd}}+\frac{N}{2}S_{0,\mathrm{even}}+2\sum_{l=1}^{N/2}[N-(2l-1)]S_{2l-1}^{eo}+2\sum_{l=1}^{N/2-1}\left[\frac{N}{2}-l\right](S_{2l}^{oo}+S_{2l}^{ee})\right]\right\}\right]^{1/2}. \tag{4}$$

The correlation scattering factors in the binary multislab medium defined by

$$S_{2\Delta-1}^{eo}\left[\frac{d}{\sigma}\right]=A\frac{\sqrt{\pi}}{2}\sigma^2\left[i\,\mathrm{erfc}\left[\tfrac{3}{4}[(2\Delta-1)+1]\frac{d}{\sigma}\right]+i\,\mathrm{erfc}\left[\tfrac{3}{4}[(2\Delta-1)-1]\frac{d}{\sigma}\right]-i\,\mathrm{erfc}\left[\tfrac{3}{4}[(2\Delta-1)+\tfrac{1}{3}]\frac{d}{\sigma}\right]-i\,\mathrm{erfc}\left[\tfrac{3}{4}[(2\Delta-1)-\tfrac{1}{3}]\frac{d}{\sigma}\right]\right],$$

$$S_{2\Delta}^{oo}\left[\frac{d}{\sigma}\right]=A_1\frac{\sqrt{\pi}}{2}\sigma^2\left[i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta+\tfrac{4}{3})\frac{d}{\sigma}\right]+i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta-\tfrac{4}{3})\frac{d}{\sigma}\right]-2i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta)\frac{d}{\sigma}\right]\right],$$

$$S_{2\Delta}^{ee}\left[\frac{d}{\sigma}\right]=A_2\frac{\sqrt{\pi}}{2}\sigma^2\left[i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta+\tfrac{2}{3})\frac{d}{\sigma}\right]+i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta-\tfrac{2}{3})\frac{d}{\sigma}\right]-2i\,\mathrm{erfc}\left[\tfrac{3}{4}(2\Delta)\frac{d}{\sigma}\right]\right], \tag{5}$$

$$S_{0,\mathrm{odd}}\left[\frac{d}{\sigma}\right]=A_1\frac{\sqrt{\pi}}{2}\sigma^2\left[i\,\mathrm{erfc}\left[\frac{d}{\sigma}\right]+i\,\mathrm{erfc}\left[-\frac{d}{\sigma}\right]-2i\,\mathrm{erfc}(0)\right],$$

$$S_{0,\mathrm{even}}\left[\frac{d}{\sigma}\right]=A_2\frac{\sqrt{\pi}}{2}\sigma^2\left[i\,\mathrm{erfc}\left[\frac{1}{2}\frac{d}{\sigma}\right]+i\,\mathrm{erfc}\left[-\frac{1}{2}\frac{d}{\sigma}\right]-2i\,\mathrm{erfc}(0)\right],$$

where the function $i\,\mathrm{erfc}(z)$ is the integral of the complementary error function [12] and A, A_1, and A_2 are the correlation amplitudes in the binary inhomogeneous dielectric medium. From the first to the third equation in Eqs. (5), they represent the cross correlations of intensity among the odd and even number slabs, respectively, at different sites of the plane of the propagation. The fourth and fifth equations represent the self-correlations of the intensity among the same layers at the sites of the perpendicular plane of the propagation. The scattering factor

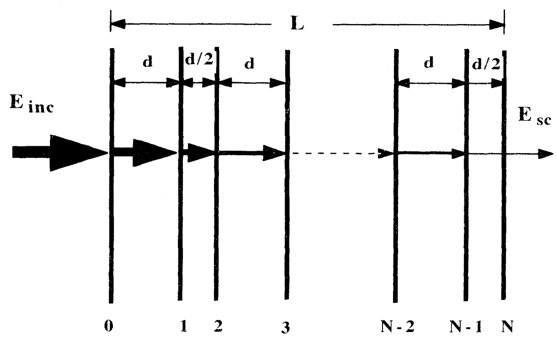

FIG. 1. Propagation through an inhomogeneous binary multislab dielectric medium. Forward scattering process in the multislab medium in the sense of line-of-sight propagation. E_{inc} an E_{sc} represent incident electric field and scattered electric field, respectively. In a binary multislab dielectric medium, the thickness is represented by d for the odd number of the slab and $d/2$ for the even number of the slab.

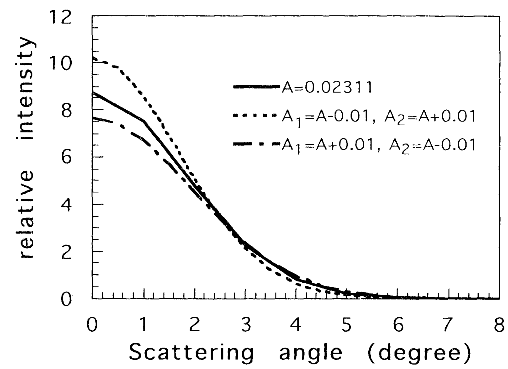

FIG. 3. Intensity profiles of the forward scattering as a function of scattering angle in the binary medium at different correlation amplitudes. We reduced the intensity scale by factor of 10^4.

represents the strength of correlations in the multislab medium and the first equation of $S^{eo}_{2\Delta-1}(d/\sigma)$ in Eqs. (5) is dominant throughout the whole medium.

In the region $\rho \gg \sigma$, the FWHM, Γ_F, of the forward scattered intensity in the multislab medium is $\Gamma_F = 2\sin^{-1}[(1/k_0 W)\sqrt{2\ln 2}]$, which is a limit of the homogeneous medium. Again, the intensity fluctuations due to the inhomogeneity are strongly correlated in the region $\rho \ll \sigma$, but the light scattering is regarded as a homogeneous random field in the region $\rho \gg \sigma$.

In Figs. 2 and 3, we used the same parameters $A = 0.02311$, $W = 1.5$ mm, $\langle \varepsilon \rangle = 3$, and $\lambda = 4880$ Å as the values of the polycrystalline samples [6] for our numerical calculations but used two different sets of parameters for A_1 and A_2 in the binary multislab medium. We use the first set of correlation amplitudes for $A_1 = A - 0.01$ and $A_2 = A + 0.01$, then we use arbitrarily the other set of parameters for $A_1 = A + 0.01$ and $A_2 = A - 0.01$. The set of parameters of the correlation amplitude may be adjusted properly if we measure the local area dielectric constant for the binary mixture samples.

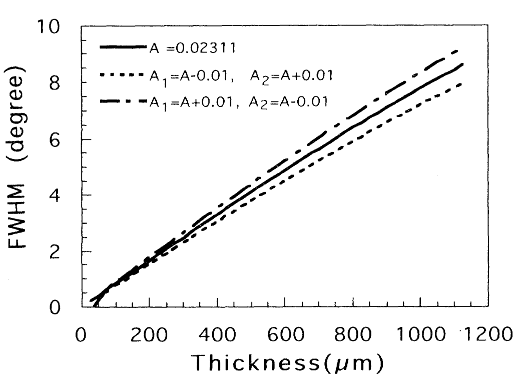

FIG. 2. Plot for the full width at half maximum (FWHM) of the forward scattering vs thickness in the binary dielectric multislab medium at different correlation amplitudes.

In Fig. 2, we present the theoretical FWHM profiles in both the binary and the single thickness discrete multislab media as a function of the thickness, with the correlation length $\sigma = 1.0$ mm. The FWHM does not sharply depend on the correlation length but it does depend on the local correlation amplitude. Thus, the structural change from the single size multislab medium to the binary multislab medium may be characterized by adjusting the local correlation amplitude. The FWHM and the intensity fluctuations from the scattering media can be determined by the strength of the scattering factors in the inhomogeneous media. In the binary multislab dielectric medium, the ratios of the scattering factors are $(S_{0,\text{odd}} + S_{0,\text{even}})/S^{eo}_{2\Delta-1} = 0.47$, $S^{oo}_{2\Delta}/S^{eo}_{2\Delta-1} = 0.28$, $S^{ee}_{2\Delta}/S^{eo}_{2\Delta-1} = 0.17$ for using $A = 0.02311$, $A_1 = 0.01311$, and $A_2 = 0.03311$, and $(S_{0,\text{odd}} + S_{0,\text{even}})/S^{eo}_{2\Delta-1} = 0.79$, $S^{oo}_{2\Delta}/S^{eo}_{2\Delta-1} = 0.72$, $S^{ee}_{2\Delta}/S^{eo}_{2\Delta-1} = 0.068$ for using $A = 0.02311$, $A_1 = 0.03311$, and $A_2 = 0.01311$. Therefore, the FWHM and intensity fluctuation can be determined dominantly by the scattering factor from the cross correlations between the thick and thin inhomogeneous layers. The FWHM is broadened if the local correlation amplitude is increased, especially, in the thick layer of the medium.

In Fig. 3, we plot the intensity distribution as a function of scattering angle with the sample thickness of 525 μm. The binary multislab thickness was taken for the first slab to be 25 μm, 12.5 μm for the second one, and so on. The intensity decreases as the scattering angle or the thickness of the medium increases. These phenomena can be understood manifestly, since the absorbance is closely related to the correlation amplitude and the scattering factor [6]. We, therefore, may apply the results of our model to obtain useful information not only for the compound materials such as the birefringent liquid crystal, the polycrystalline, and the cementum but also for the human body by applying the proper wavelength and power of the incident source.

To conclude, we find that the FWHM and intensity fluctuation can be determined dominantly by the scattering factor from the cross correlations between the thick and thin inhomogeneous layers. The FWHM is broadened if the local correlation amplitude is increased, especially in the thick layer of the medium. The forward scattering intensity is characterized in a binary type of the dielectric inhomogeneous layers compared to the identical size of the thickness multislab inhomogeneous medium.

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